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B.A./B.Sc. (Sem. I) Examination, 2023-24

(राष्ट्रीय शिक्षा नीति - 2020)

(मेजर/माइनर)

MATHEMATICS

Paper Code - B030101T

(Differential Calculus & Integral Calculus)

Time : Two Hours] *[Maximum Marks : 75*

Note : Attempt **all** sections as per instructions.

Section - A

Note : Attempt **all** questions. Each question carries 3 marks. 3×10=30

1. (a) Define directional derivative of a function and give an example also.

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ find div $\vec{r}$

(c) Write statement of Stokes theorem.

(d) If ϕ is a scalar function, find curl grad ϕ.

(e) Define rectification of a curve.

P.T.O.

(f) Show that B(m,n) is symmetric in the indices's m and n.

(g) State Pappus theorem.

(h) Find the asymptote of the curve $x^2y^2 = a^2(x^2+y^2)$.

(i) Define envelope, give an example also.

(j) What is a removable singularity, explain with an example.

Section-B

Note : Attempt any **four** questions. Each question carries 6 marks. 6×4=24

2. Verify Stoke's theorem for the function $\vec{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$ taken around curve bounded by x = ± a, y=0, y= b.

3. Evaluate

$$\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

4. Prove that

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

where m and n are positive.

5. Change the order of integration in the integral
$$\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\, dx\, dy$$
and hence find its value.

6. Test the convergence on divergence of following series
$$x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \frac{15}{17}x^4 + \ldots\ldots$$

7. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$
show that
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$

8. If $y = a\cos(\log x) + b\sin(\log x)$
show that $x^2y_2 + xy_1 + y = 0$ and hence show that
$$x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

Section-C

Note : Answer any **two** questions. Each question carries **10½** marks.

10½×2=21

9. Find the constants a, b, c so that the vector
$$\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$
is irrotational.

10. Transform the integral
$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\, dx\, dy}{\sqrt{x^2+y^2}}$$
by changing to polar coordinates and hence evaluate it.

11. A function f(x) is defined by
$$f(x) = \begin{cases} x & \text{if } x < 1 \\ 2 - x & \text{if } 1 \le x \le 2 \\ -x^2 + 3x - 2 & \text{if } x > 2 \end{cases}$$
Check the continuity and differentiability of f(x) at x=1 and x=2.

12. Evaluate the following limits

(i) $\lim_{x \to 0} \frac{\log\{\log(1-x^2)\}}{\log\{\log(\cos x)\}}$

(ii) $\lim \left\{\frac{\tan x}{x}\right\}^{1/x^2}$

(iii) $\lim \left\{\frac{x - \tan x}{x^3}\right\}$

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