J (Printed Pages 7)

1215

B.A./B.Sc. (Part-II) Examination, 2022

MATHEMATICS

Paper - I

(Linear Algebra and Matrics)

Time : Three Hours] *[Maximum Marks : 65*

Note : Attempt **all** sections as per instructions.

Section-A

Note : Attempt **all** questions. Each question carries one mark. 1×10=10

1. (i) Show that $W=\{(x,y,z): x,y,z$ are rationals$\}$ is not a subspace of $R^3(R)$.

(ii) Define basis of a vector space.

(iii) Show that the map $T:R^2\rightarrow R^2$ give by $T(x,y) = (y,x)$ is linear.

P.T.O.

(iv) Define nullity of a linear transformation.

(v) Let V be an inner product space over F then prove that $\|Kx\| = |K|\|x\|$, $\forall x\in V$ and $\forall K\in F$.

(vi) Define symmetric and skew symmetric bilinear forms.

(vii) Prove that the matrix $\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$ is unitary.

(viii) Let A be any complex square matrix. Prove that $H= \overline{A}^t A$ is hermitian.

(ix) What is the condition for consistency of a non-homogeneous system of linear equation.

(x) Determine the eigen values of the matrix $A = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$

Section-B

Note : Attempt **all** questions. Each question carries 7 marks. $7 \times 5 = 35$

2. Prove that the union of two subspaces of a vector space is a subspace if and only if one is contained in the other.

OR

Find a basis and dimension of subspaces W of $R^3(R)$ where :

(a) $W = \{(x,y,z) : x+y+z = 0\}$

(b) $W = \{(x,y,z) : x = y = z\}$

3. Prove that the vector space V is the direct sum of its subspaces W_1 and W_2 if and only if

(i) $V = W_1 + W_2$

(ii) $W_1 \cap W_2 = \{0\}$

OR

Find a linear map $T : R^3 \to R^4$ whose image is generated by $\{(1,2,0,-4), (2,0,-1,-3)\}$.

4. Let V(F) is a vector space of all polynomials in x of degree ≤ 3. Find the matrix of the differentiation transformation $D : V \to V$ defined as $D(f(x)) = \frac{df(x)}{dx}$ relative to the basis $B = \{1, x, x^2, x^3\}$.

OR

If the set $\{x_1, x_2, \ldots\ldots x_n\}$ is orthonormal in an inner product space V, then prove that for any $x \in V$, the vector

$y = x - \langle x, x_1\rangle x_1 - \langle x, x_2\rangle x_2 - \ldots - \langle x, x_r\rangle x_r$ is orthogonal to each x_i.

5. Which of the following functions f defined on vectors $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$ in R^2 are bilinear forms?

(i) $f(\alpha,\beta) = x_1y_2 - x_2y_1$

(ii) $f(\alpha,\beta) = (x_1 - y_1)^2 + x_2y_2$

OR

Reduce the following matrix in normal form and hence find the rank :

$$\begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

6. Find λ and μ so that the following equation are

(i) In consistent

(ii) Consistent with unique solution

(iii) Consistent with infinite number of

solutions

$$x+2y+3z = 10$$

$$x+y+z = 6$$

$$x+2y+\lambda z = \mu$$

OR

Prove that the eigen values of a Hermitian matrix are real.

Section-C

Note : Attempt any **two** questions. Each question carries 10 marks. $10\times2=20$

7. Prove that every linearly independent subset of a finitely generated vector space V(F) can be extended to form a basis.

8. State and prove Rank-Nullity Theorem.

9. Find the dual basis of the basis B={(1,−1,3), (0,1,−1), (0,3,−2)} for $V_3(R)$.

10. State and prove Cayley-Hamilton theorem.

11. Show that the matrix

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

is diagonalizable. Also find the diagonal form and a diagalizing matrix P.