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19/185-B

B.A./B.Sc. (Part-III) Examination, 2019

MATHEMATICS

First Paper

BMG - 301

(Real Analysis)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from **all** sections as per instructions.

Section - A

(Very Short Answer Type Questions)

Note : Attempt **all** parts of this question. Give answer of each part in about 50 words.

1½×10=15

1. (a) Show that every subset of a countable set is countable.

   (b) Show that the set N of natural numbers has no limit points.

   (c) Define convergent and divergent sequences with one example.

   (d) Show that the sequence $\left\langle \frac{1}{n} \right\rangle$ has the limit 0.

P.T.O.

   (e) State Taylor's theorem for functions of two variables.

   (f) If f is uniformly continuous on an interval I, then prove that it is continuous on I.

   (g) Discuss the convergence of

$$\int_1^\infty \frac{dx}{\sqrt{x}}$$

   (h) Evaluate $\int_0^1 \frac{dx}{\sqrt{x}}$

   (i) Define metric space.

   (j) Show that in a discrete metric space, every set is open.

Section - B

(Short Answer Type Questions)

Note : Attempt **all** questions. Give answer of each question in about 200 words. 8×5=40

2. Let a be any real number and b any positive real number, then prove that there exists a positive integer n such that

   $nb > a$

   OR

   If N is the set of natural numbers prove that N×N is countable.

3. Prove that a sequence of real numbers converges

if and only if it is a Cauchy sequence.

OR

Prove that :

$$\lim\left[\left(\frac{2}{1}\right)\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3 \cdots\cdots\left(\frac{n+1}{n}\right)^n\right]^{1/n}=0$$

4. If a function f is continuous in the closed interval [a, b], Then prove that f(x) must take at least once all values between f(a) and f(b).

OR

Discuss the continuity of the function

$$f(x,y) = \frac{2xy^2}{x^3+3y^3}, \quad (x,y)\neq(0,0)$$

and f(0,0) = 0 with respect to both variables.

5. Prove that the lower R-integral can not exceed the upper R-integral.

OR

Test the convergence of the integral

$$\int_0^\infty e^{-x^2}\,dx$$

6. Prove that in a metric space, every closed sphere is a closed set.

OR

Prove that in a metric space (x,d), $\bar{A} = A \cup D(A)$ where A is a subset of metric space.

Section - C

(Long Answer Type Questions)

Note : Attempt any **two** questions. Give answer of each question in about 500 words. 10×2=20

7. Prove that every infinite bounded set of real numbers has a limit point.

8. Test for uniform convergence the series

$$\sum_{n=0}^{\infty} x e^{-nx}$$

in the closed interval [0,1]

9. Find the first six terms of the expansion of the function $e^x \log(1+y)$ in a Taylor's series in the neighbourhood of the point (0,0).

10. If $f \in R[a,b]$ then prove that $|f| \in R[a,b]$ and

$$\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$$

11. Prove that every convergent sequence in a metric space is a Cauchy sequence. Is the converse of this theorem true? Give an example in support of your answer.