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6245

B.A./B.Sc. (Part-III) Examination, 2021

MATHEMATICS

Paper - I

(Real Analysis)

Time : 1½ Hours] *[Maximum Marks : 60*

Note : Attempt **all** sections as per instructions.

Section-A

Note : Attempt any **five** parts of this question. Each question carries 4 marks. 5×4=20

1. (i) Prove that, for any real number x,there exists a unique integer 'n' such that $n \le x < n+1$.

(ii) Prove that the set of all rational numbers is denumerable (or countably infinite).

P.T.O.

(iii) Show that function f(x) defined as $f(x) = |x| \ \forall\ x \in R$ is continuous at x=0 but not differentiable.

(iv) Examine the continuity at (0,0) of the function

$$f(x,y) = \frac{2xy^2}{x^3+3y^3};(x,y) \neq (0,0) \text{ and}$$

f(0,0)=0

(v) If $f: [a,b] \to R$ is bounded and P is any partition of I, then prove that L (P,f) ≤ (P,f).

(vi) Test the convergence of $\int_0^\infty e^{-mx}\,dx, m > 0$.

(vii) Show that the series $\sum_{n=1}^{\infty} x^{n-1}(1-x,) \forall x \in [0,1]$ is not uniformly convergent.

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(viii) Write the statement of Abel's Test and Dirichlet's Test for uniform convergence of series.

(ix) If x be the metric space, then show that

(I) Int $(\phi) = \phi$

(II) Int $(x) = x$

(x) Prove that any superset of a neighbourhood (nbd) of a point in the metric space x is also a neighbourhood (nbd) of that point.

Section-B

Note : Attempt any **two** questions. Each questions carries 12½ marks. 2×12½=25

2. Discuss whether the sequence $\left\{\frac{2^n}{n!}\right\}$ is monotonic, bounded and convergent.

OR

It f is continuous at a point 'a', then show that |f| is also continuous at 'a' But converse need not to be True.

3. Prove that every Cauchy sequence is bounded but the converse is not necessarily True.

OR

Show that the function f (x) = x is Riemann integrable on [0,1].

4. Examine the continuity at x=a of the function $f(x) = \begin{cases} \frac{1}{x-a} \operatorname{cosec} \frac{1}{x-a} & ; x \neq a \\ 0 & ; x = a \end{cases}$

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OR

Use Abel's Test to prove that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$ is uniformly convergent on [0,1]

5. If A is any subset of the metric space x, then prove that $\bar{A} = A \cup D(A)$.

OR

Show that the term by term differentiation of the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^3}$ is possible and also find its derivative.

6. Show that in any metric space x, each open sphere is an open set.

OR

Examine the convergence of the integral $\int_0^\infty \frac{x^{2m}}{1+x^{2n}} dx$, where m and n are positive integers.

Section-C

Note : Attempt any **one** question. Each question carries 15 marks. $1\times15=15$

7. Prove that a monotonic sequence is convergent if it is bounded.

8. Fine the maximum value of u=8xyz where $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

9. If f is bounded and Riemann integrable on [a,b] and if there is a differentiable function F on [a,b] such that F′=f, then prove that $\int_a^b f(x)dx = F(b)-(a)$.

10. If (x,d) is a metric space, then d* defined by $d^*(x,y) = \frac{d(x,y)}{1+d(x,y)}$ is also a metric on x.

11. Show that the sequence $<f_n>$ of functions defined by $f_n(x) = x^n, \forall x \in [0,1]$ is point-wise convergent. Find its limit function also. Further, show that it does not converge uniformly in [0,1].