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M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper - I

(Advanced Abstract Algebra)

Time : Three Hours] *[Maximum Marks : 100*

Note : Attempt **all** sections as per instructions.

Section-A

Instruction : Attempt **all** parts of the following. Each part carries 2 marks. 2×10=20

1. (a) Prove that Conjugacy is an equivalence relation in a group.

   (b) Show that $[Q(\sqrt{2},\sqrt{3}):Q]=4$.

   (c) Define composition series of a group and give an example.

   (d) Show that the symmetric group of degree 3 is solvable.

   (e) Differentiate between Jordan block and Jordan form.

P.T.O.

   (f) Let Q be the field of rational numbers. Show that the field $Q(\sqrt{2})=\{a+b\sqrt{2}:a,b\in Q\}$ is a finite extension of Q.

   (g) Let a∈k the algebraic over F and let p(x) be a minimal polynomial for a over F. Prove that p(x) is irreducible over F, where K is extension field of F.

   (h) Define adjunction in Field Extension.

   (i) Prove that every finite field is Algebraic.

   (j) If M is an R-module and a∈M, show that Ra={ra : r∈R} is an R-submodule of M.

Section-B

Instruction : Attempt **all** questions. Each question carries 10 marks. 10×5=50

2. Show that $a\to a^{-1}$ is an automorphism of a group G iff G is abelian.

OR

Give an example to show that in a group G the normalizer of an element is not necessarily a normal subgroup of G.

3. Let G be a group and G′ be the commutator subgroup of G. Prove that if N is any nor-

mal subgroup of G, then G/N is abelian iff $G' \subseteq N$.

OR

Prove that every abelian group of order 6 is cyclic.

4. For the matrix $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{pmatrix}$ Find all possible Jordan Forms which are canonical.

OR

If S and T are nilpotent linear transformations which commute, prove that ST and S+T are nilpotent linear transformations.

5. If L is an algebraic extension of K and K is an algebraic extension of F, prove that L is an algebraic extension of F. i.e Algebraic Extension is Transitive.

OR

Let Q be the field of rational numbers and $f(x) = x^4+x^2+1 = \in Q[x]$. Show that Q (ω), where $\omega = \frac{-1+i\sqrt{3}}{2}$ is a splitting field of f(x).

6. Let M be a free R-module with a basis con-

sisting of n elements. Prove that M is isomorphic to R^n.

OR

Let C be the field of complex numbers and R be the field of real numbers. Prove that C is a normal extension of R.

Section-C

Instruction : Attempt any **two** questions. Each question carries 15 marks. 15×2=30

7. Prove that every filed of characteristic D is Normal extension of field.
8. State and prove first sylow theorem.
9. Find the Jordan canonical form of the matrix. $\begin{pmatrix} -1 & -2 & -1 \\ -1 & -1 & -1 \\ 2 & 3 & 2 \end{pmatrix}$
10. Let K be an extension of a field F. Prove that the element $a \in K$ is a algebraic over F if and only if F(a) is a finite extension of F.
11. State and prove fundamental theorem of Homo morphisms.