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M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper - II

(Advanced Real Analysis and Measure Theory)

Time : Three Hours] *[Maximum Marks : 100*

Note : Attempt **all** sections as per instructions.

Section - A

Note : Attempt **all** parts of this question. Each part carries 2 marks. 2x10=20

1. (a) Write the condition under which f(x) is Riemann-Stieltjes integrable.

   (b) Show that every real function of bounded variation on [a,b] is bounded.

   (c) Can the brackets be removed from the series

   $$\left(1-\frac{1}{2}\right)+\left(1-\frac{3}{4}\right)+\left(1-\frac{7}{8}\right)+.....$$

P.T.O.

   (d) Find the value of $\int_0^1 x^3 d(x^2)$

   (e) Define measurable set.

   (f) Give a example of uncountable set of measure zero.

   (g) Show that every constant function is measurable.

   (h) Define $||f||_p$ for $f \in L^p(a,b)$

   (i) State Jonsen Inequality.

   (j) Define downward convex function f(x).

Section - B

Note : Attempt **all** questions. Each question carries 10 marks. 5×10=50

2. If f is continuous and g is monotonically non-decreasing on [a, b]

   Then $f \in RS(g)$.

   OR

   If $f \in RS(g)$ and $|f(x)| \le k$ on $[a,b]$ then prove that $\left|\int_a^b f dg\right| \le k(g(b)-g(a))$

3. Suitably re-arranging the series

$S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + ...$

prove that $S = \frac{1}{2}S$. Explain Fallacy.

OR

If f and g are functions of bounded variation on [a, b] then prove that f+g is also function of bounded variation on [a, b].

4. For every set A prove that $m_*A \leqslant M^*A$ where m_* stands for lower measure and m^* stands for outer measure.

OR

If $m^*E=0$, then E is measurable.

5. Prove that every continuous function is measurable.

OR

Show that a countable set is measurable and its measure is zero.

6. If $f \in L^p$ and $g \leqslant f$ then $g \in L^p$.

OR

If f and g are bounded function and Lebesgue integrable over [a, b] and if $f(x) \geqslant 0$ a.e on [a, b] then $\int_a^b f(x)dx \geqslant 0$.

Section - C

Note : Attempt any **two** questions. Each question carries 15 marks. 2×15=30

7. If f is R-integrable on [a, b] and g is monotonically non-decreasing function on [a, b] such that g′ is R-integrable on [a, b] then $f \in RS[a,b]$ and $\int_a^b f dg = \int_a^b f(x)\, g'(x)\, dx$.

8. If f and g be functions of bounded variation on [a, b] and f is also continuous on [a, b] then $\int_a^b f\, dg = f(b)g(b) - f(a)g(a) - \int_a^b g\, df$

9. Show that $\int_0^3 x\, d\{[x] - x\} = \frac{3}{2}$.

10. Every bounded function defined on [a, b] is Lebesgue integrable.

11. If $f \in L^p[a, b]$, $g \in L^q[a, b]$ for p>1 where $L^q[a, b]$ is conjugate of $L^p[a, b]$ then show that $fg \in L^p[a, b]$ and $||fg|| \leq ||f||_p\, ||g||_q$