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(Printed Pages 7)

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M.A./M.Sc. (Final) Examination, 2022

Mathematics (Optional Paper)

Paper - V

((b) Advanced Riemannian Geometry)

Time : Three Hours] *[Maximum Marks : 100*

Note : Attempt **all** sections as per instructions.

Section-A

Instruction : Attempt **all** parts of the following question. Each part carries 2 marks. 2×10=20

1. (a) Define normal curvature of a hyper-surface of a Riemannian space in the direction of a curve C.

P.T.O.

(b) Define principal direction and line of curvature in a hypersurface of a Riemannian space.

(c) Define an asymptotic direction and an asymptotic line in a hypersurface of a Riemannian space.

(d) Write Dupin's theorem for a subspace of a Riemannian space.

(e) What are necessary and sufficient conditions for a subspace V_n to be totally geodesic with respect to an enveloping spave V_m?

(f) Derive Lie-derivative of a contravariant vector.

(g) When is a subspace V_n said to be a minimal variety for an enveloping space V_m?

(h) Write Lie derivative of the Christoffel symbol of second kind in the tensorial form.

(i) Prove that $a_{\alpha\beta; i}=0$

(j) Define a Hyper sphere in Eucledeam space.

Section-B

Instruction : Attempt **all** questions of this section Each questions carries 10 marks. $10\times5=50$

2. Define a subspace V_n of a Riemannian space V_m (n<m). Show that there are (m−n) linearly independent vector fields normal to V_n.

OR

Derive Weingasten's formulas for a hypersurface of a Riemannian space.

3. Derive Euler's formula for a hypersurface of a Riemannian space.

OR

Prove that a totally geodesic hypersurface is a minimal hypersurface, and its lines of curvature are in determinate.

4. Derive Gauss's formulas for a subspace of a Riemannian space.

OR

Prove that the derived vector of the unit normal with respect to the enveloping space, along a curve in the hypersurface, will be tangential to the curve provided it be a line of curvature of the hypersurface.

5. Prove that tendency of a unit normal $N^{\alpha}_{v|}$ to a subspace V_n of V_m in the direction of a curve c is given by $-\Omega_{v|ij}\frac{dx^i}{ds}\frac{dx^j}{ds}$

OR

Find the Lie derivative of a covariant vector field Bj in a Riemannian space.

6. Prove that any two distinct principal directions relative to a normal $N^{\alpha}_{v|}$ in the nnd of a point O are mutually orthogonal.

OR

Find the condition for orthogonal intersection of the hyperspheres

$$\sum_i[(y^i)^2-2c^iy^i]+k=0$$

and $\sum_i[(y^i)^2-2\bar{c}^iy^i]+\bar{k}=0$

Section-C

Instruction : Attempt **any two** questions of this section. Each question carries 15 marks. 2×15=30

7. Let V_n, equipped with coordinates x^i, be a subspace of a Riemannian space V_m,

P.T.O.

equipped with coordinates y^α. Let $A^\alpha_{\beta i}$ be a mixed tensor of second order in the y's and covariant vector in the x's. Find the tensor derivative of $A^\alpha_{\beta i}$ with respect to x^j.

8. Prove that for a hypersurface of a space of constant curvature K, the characteristic equations of Gauss and those of Mainardi-codazzi are respectively

$$R_{hijk}=(\Omega_{hj}\,\Omega_{ik}-\Omega_{hk}\,\Omega_{ij})+K(g_{hj}\,h_{ik}-g_{hk}\,g_{ij})$$

and $\Omega_{ij},k-\Omega_{ik},j=0.$

9. For a curve C in a subspace V_n of a Riemannian space V_m, prove that

$$K_a^2=k_n^2+K_g^2,$$

where symbols have their usual meaning.

10. Define motion in a Riemannian space. Prove that necessary and sufficient conditions for a Riemannian space to admit a motion is

that the Lie derivative of its metric tensor vanishes.

11. Define affine motion. Prove that necessay and sufficient condition for existence of affine motion in a Riemannian space is that

$$D_L\,\Gamma^i_{jk}=0$$