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M.A./M.Sc. (Final) Examination, 2022

MATHEMATICS

Paper - II

(Dynamics of Rigid Bodies & Analytical Dynamics)

Time : Three Hours] *[Maximum Marks : 100*

Note : Attempt **all** sections as per instructions.

Section-A

Note : Attempt **all** questions. Give answer of each question in about 50 words. Each question carries 2 marks. 2×10=20

1. (a) Define radius of gyration.

(b) Find moment of inertia of a circular disc of radius a about its diameters.

(c) Define impulsive forces.

(d) Define D' Alembert Principle of Motion.

(e) State principle of conservation of energy.

(f) Define degree of freedom.

(g) Define holonomic and non-holonomic system.

(h) Show that the transformation

$q = \sqrt{(2P)}\sin Q, p = \sqrt{(2P)}\cos Q$ is canonical.

(i) Prove that the transformation Q= q tan p, P= log sin p is canonical.

(j) Define Lagrange bracket.

Section-B

Note : Attempt **all** questions. Give answer of each question in about 200 words. Each question carries 10 marks. 5×10=50

2. Show that for a thin hemi-spherical shell of mass M and radius a the moment of inertia about any line through the vertex is $\frac{2}{3}Ma^2$.

OR

Show that the momental ellipsoid at a point on the edge of the circular base of a thin hemi-spherical shell is

$2x^2+5(y^2+z^2)-3zx=$ constant.

3. A plank of mass m and length 2a, is initially at rest along a line greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M, starting from the upper end walks downs the plank so that it does not move. Show that he will

reach the other end in time

$$\left[\frac{4Ma}{(m+M)g\sin\alpha}\right]^{\frac{1}{2}}$$

OR

Derive Euler's Equations in rigid dynamices.

4. Derive equation of motion relative to centre of Inertia.

OR

Prove that when a body moves under the action of a system of conservative forces the sum of its kinetic and potential energies is constant throughout the motion.

5. Find the equation of motion of one-dimensional harmonic oscillator using Hamilton's principle.

OR

Derive Lagrange Equations for Holonomic dynamical systems.

6. Find the Hamiltonian and Hamilton's equations for a charged particle in an electromagnetic field.

OR

Find the equation of motion of a simple pendulum using Lagrange Equations.

Section-C

Note : Attempt any **two** questions. Give answer of each question in about 500 words. Each question carries 15 marks. 15×2=30

7. Find the moment of inertia about the x-axis of the portion of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ which lies in the positive octant, supposing the law of volume density to be $\rho=\mu xyz$.

8. A uniform rod of length 2a, is placed with one end in contact with a smooth horizontal table and is then allowed to fall, if α be its initial inclination to the verticlal. Show that its angular velocity when it is inclined at an angle θ is

$$\left[\frac{bg}{a}\cdot\frac{\cos\alpha-\cos\theta}{1+3\sin^2\theta}\right]^{\frac{1}{2}}$$

. Find also the reaction of the table.

9. Derive Lagrange Equation from Hamilton's Equation of motion.

10. Derive Jacobi-Hamilton Equation.

11. Find Moment of inertia of a body about a line in three dimension.